Elastic scattering in 2D.

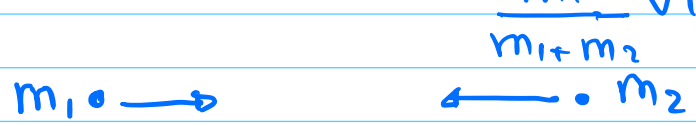
Strength of cm frame.



$$V_{cm} = \frac{m_1 v_i + m_2 0}{m_1 + m_2} =$$

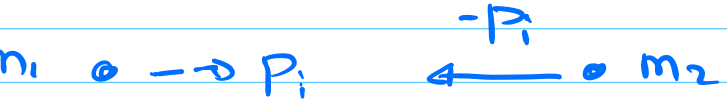
$$= \frac{m_1}{m_1 + m_2} v_i$$

$\Rightarrow$ in cm:



$$v_i - \frac{m_1}{m_1+m_2} v_i = \frac{m_2}{m_1+m_2} v_i$$

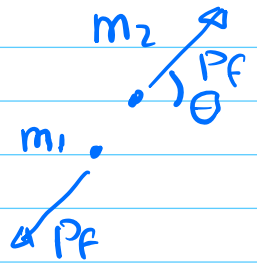
$\Rightarrow$



$$p_i = \frac{m_1 m_2}{m_1 + m_2} v_i$$

$\Rightarrow p_1 + p_2 = 0$ in cm.

After collision:



Look at kinetic energy in cm:

$$k_i = \frac{p_i^2}{2m_1} + \frac{p_i^2}{2m_2} =$$

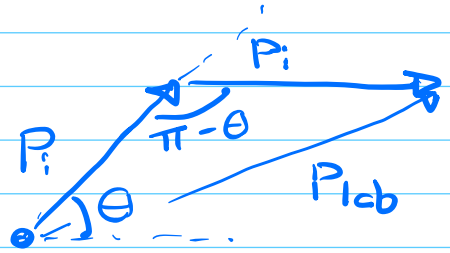
$$= \frac{1}{2} \frac{m_1 m_2}{m_1 + m_2} p_i^2$$

$$k_f = \frac{1}{2} \frac{m_1 m_2}{m_1 + m_2} p_f^2$$

Elastic scattering $\Rightarrow k_f = k_i$

$\Rightarrow p_f = p_i$

Now look at m_2 and go back to the lab frame:

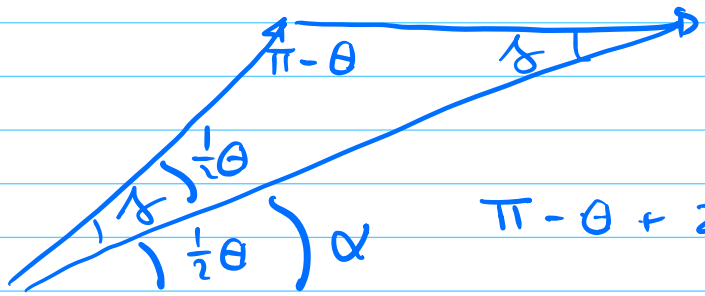


$$p_{lab}^2 = p_i^2 + p_i^2 - 2p_i^2 \cos(\pi - \theta) :$$

$$= 2(1 + \cos\theta) p_i^2$$

$\Rightarrow$ can determine $k_{2,f,lab}$

Now look at triangle:



$$\pi - \theta + 2\delta = \pi$$

$$\Rightarrow \delta = \tfrac{1}{2}\theta$$

α = lab scattering angle of m_2

$$\alpha = \tfrac{1}{2}\theta_{cm}$$

↑ scattering angle in cm.